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Nonlinear Dynamic Response of Shells of Revolution

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ABSTRACT: The nonlinear dynamic buckling behavior of axisymmetric spherical shells subjected to radial dynamic edge loading has been investigated. The peripheral dynamic loading is in the form of step loading of infinite duration in the time domain. This kind of loading is seen in actuators known as displacement transducers, where the radial motion of the edges is converted into a flex-tensional motion in the spherical caps. As a result a large displacement is obtained in the perpendicular direction, which may result in snap-through buckling. It is theoretically possible to develop magnification mechanisms that produce sizeable displacements necessary for actuating functions. For the numerical solution of the problem a computer program, using a linearized finite element incremental-iterative approach based on updated Lagrangian formulation is developed, and the whole process is accomplished using the Newmark b method as the time integration scheme.

Keywords: Nonlinear analysis; peripheral loading; snap-through buckling; finite elements

ÖZET: Lineer olmayan statik ve dinamik analizler için kademeli arttırımlı ardışık yöntemler kullanarak sonlu elemanlar bilgisayar programı hazırlanmıştır. Yüklerin kenar deplasmanları olarak uygulanmasının ilginç örneği, metal-seramik kompozitlerde rastlanmaktadır. Seramikteki yatay deplasmanların doğuracağı hareket dikeyine büyük deplasmanlara, o da basık kabukta vurgu instabilitesine neden olabilmektedir. Elde edilen sonuçlar yukarıda bahsedilen aletlerin tasarımda uygulandığında mekanizma sonucu büyüyen deplasmanların hesaplanması gerekecektir. Sayısal çalışmada burkulma durumunda ileri atlamanın mümkün olduğu, dinamik yükler altında incelenmiştir. Dinamik yükler, zaman uzayında sonsuz süreli basamak yükü olarak uygulanmıştır.

Anahtar kelimeler: Lineer olmayan analiz, vurgu instabilitesi, sonlu elemanlar, kenar yükleme

Introduction

The nonlinear dynamic analysis of shells of revolution has been the topic of a large number of investigations since early sixties. Recent years have seen significant advances in the technology of displacement actuators (Sugawara, Onitsuka, Yoshikawa and Newnham, 1992). These devices are usually in the form of axisymmetric shallow spherical shells and the main loading is applied at the edges. The nonlinear axisymmetric snap-through buckling characteristics of the shells play an extremely important factor in the design. The static buckling behavior of axisymmetric shells subjected to axisymmetric peripheral edge load or displacement for various shell parameters and boundary conditions were reported previously (Akkas and Odeh, 1998, 2000, 2001).

Finite Element Model and Formulation

The finite elements used are one-dimensional conical frustra. Two-nodded conical frustra proved to be very efficient when applied to thin shells. The results obtained using these elements are in satisfactorily agreement with those achieved with more complex finite elements. The shear effect is neglected because the shell is assumed to be thin. A direct finite element approach using displacement function is applied. The number of elements used in the radial direction is 20. A comparison of results obtained from models with 40 elements reveals that 20 elements were sufficient for numerical accuracy. The material used is assumed to be linearly elastic. The computer program developed uses a linearized finite element incremental-iterative approach based on updated Lagrangian formulation (Bathe, 1982).

Geometric Nonlinearity

In nonlinear analysis the equilibrium of the body, needs to be established in the current configuration. Thus, an incremental formulation is essential in describing the loading and motion of the body. An extension to permit nonlinear geometric behavior has been included by adding a term to the meridian strain given in Eq.(1) to obtain

$$e_s = \frac{du}{ds} + \frac{1}{2}\left(\frac{dw}{ds}\right)^2 \tag{1}$$

The equation of motion of the finite element method is

$$M\ddot{a} + Ka = F \tag{2}$$

where M and K are the system mass and stiffness matrix, and a and F are the nodal displacements and nodal forces. This equation is satisfied at any instant of time t. Assume that the time increment Δt is small enough and the differences of the stiffness matrix K between time t and time $t+\Delta t$ is negligible. Then the finite element linear incremental equation becoms

$$M\Delta\ddot{a}^t + K^t\Delta a^t = \Delta F^t \tag{3}$$

where $\Delta\ddot{a}^{t} = \ddot{a}^{t+\Delta t} - \ddot{a}^{t}$; $\Delta a^{t} = a^{t+\Delta t} - a^{t}$ and $\Delta F^{t} = F^{t+\Delta t} - F^{t}$

The equilibrium conditions between internal and external forces must be satisfied (Zienkiewics, 1991).

$$\Psi(a) = \int_V \overline{B}^T s \; dV - F = 0 \tag{4}$$

where Ψ represnts the sum of external and internal forces and $\overline{B}$ is defined as

$$e = \overline{B}a \quad \text{where} \quad \overline{B} = B_0 + B_L(a) \tag{5}$$

in which B_0 is the same matrix as in linear infinitesimal strain analysis and only B_L depends on the displacement. In general B_L is a linear function of such displacements. If strains are small, then

$$s = De \tag{6}$$

Solution processes

The relation between δa and $\delta\Psi$ has to be found by taking the appropriate variations of Eq.(4) with respect to δa we have

$$d\Psi = \int_V d\,\overline{B}^T s \; dV + \int_V \overline{B}^T ds \; dV = K_T da \tag{7}$$

and using Eqs.(5) and (6) we have

$$ds = Dde = D\overline{B}da \;\text{ and }\; d\overline{B} = dB_L \tag{8}$$

Therefore

$$d\Psi = \int_V dB_L^T s \; dV + \overline{K}da \tag{9}$$

where

$$\overline{K} = \int_V \overline{B}^T D \overline{B} \, dV = K_0 + K_L \tag{10}$$

K_0 is the small displacements stiffness matrix, i.e.

$$K_0 = \int_V B_0^T D \, B_0 dV \tag{11}$$

K_L is the large displacement matrix and contains linear and quadratic terms in a

$$K_L = \int_V (B_0^T D \, B_L + B_L^T D \, B_L + B_L^T D \, B_0) \, dV \tag{12}$$

$$\int_V dB_L^T s \; dV \equiv K_s da \tag{13}$$

where K_s is the *geometric matrix* that depends on the stress level. Thus,

$$d\Psi = (K_0 + K_s + K_L)da = K_T da \tag{14}$$

where K_T is the total, *tangential stiffness,* matrix. In addition to K_0 of the linear theory, we need to calculate K_L and K_s when dealing with the buckling of shells. K_L is a nonlinear matrix and is a function of displacement. If the coordinate system during each time step is redefined then the initial displacement becomes a negligibly small value so that this matrix can be omitted on the basis of its high-order small value.

$$M\Delta\ddot{a}^t + (K_0^t + K_s^t + K_L^t)\Delta a^t = \Delta F^t \tag{15}$$

Solution using the Mewmark algorithm:

$$\Psi(\ddot{a}_{n+1}) = M\ddot{a}_{n+1} + P(\bar{a}_{n+1} + b\Delta t^2\ddot{a}_{n+1}) + F_{n+1} = 0 \tag{16}$$

where $\quad \bar{a}_{n+1} = a_n + \Delta t\dot{a}_n + \frac{1}{2}\Delta t^2\ddot{a}_n\ ; \quad \bar{\dot{a}}_{n+1} = \dot{a}_n + \Delta t\ddot{a}_n\ ; \quad \bar{\ddot{a}}_{n+1} = \ddot{a}_n$

and $\quad a_{n+1} = \bar{a}_{n+1} + b\Delta t^2(\ddot{a}_{n+1} - \ddot{a}_n)\ ; \quad \dot{a}_{n+1} = \bar{\dot{a}}_{n+1} + g\Delta t(\ddot{a}_{n+1} - \ddot{a}_n)$

P is a vector of nonlinear internal forces. The standard Newmark parameters are $b = 0.25$ and $g = 0.5$

Dynamic snap-through buckling of spherical shells with opening at the apex, with b = 200 mm, a = 1000 mm, R = 2500 mm and Poisson's ratio, v = 1/3 subjected to radial step load of infinite duration as shown in Fig.6 is investigated. At the hole there is a flexible ring. The boundary conditions at the hole allow rotation and vertical motion.

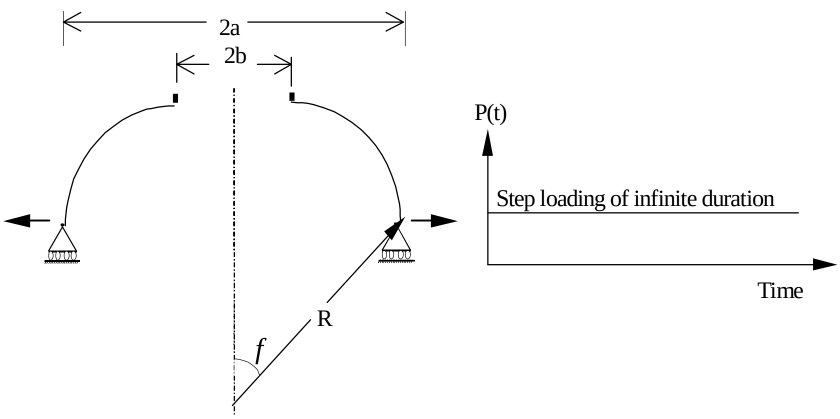

Figure 1. Spherical shell with opening at the apex subjected to step loading of infinite duration at the edge.

The step-by-step integration scheme using the Newmark b method is performed. The time increment, Δt is taken 1×10^{-4} sec. which is about $T_n/20$, where T_n is the natural period of the system. The time increment, Δt is taken sufficiently small to avoid mathematical instabilities. The duration of time is taken as 2 sec., and the problem is solved iteratively by the Newton-Raphson method. The maximum number of iterations used is 20. For load levels less than the critical load, the shell oscillates about its static equilibrium position with relatively small amplitudes. However, when the load exceeds the critical level, the oscillations of the shell become very large and the axisymmetric dynamic snap-through buckling is said to have occurred. The load which corresponds to a sudden large increase in the deflection is said to be the axisymmetric dynamic snap-through buckling load as shown in Fig. 2, where w is the maximum absolute vertical displacement and H is the height of the complete shell. The critical load versus the maximum absolute deflection of the apex curve for λ = 6 is plotted and the load at which a large increase in the deflection is demonstrated in Fig. 3.

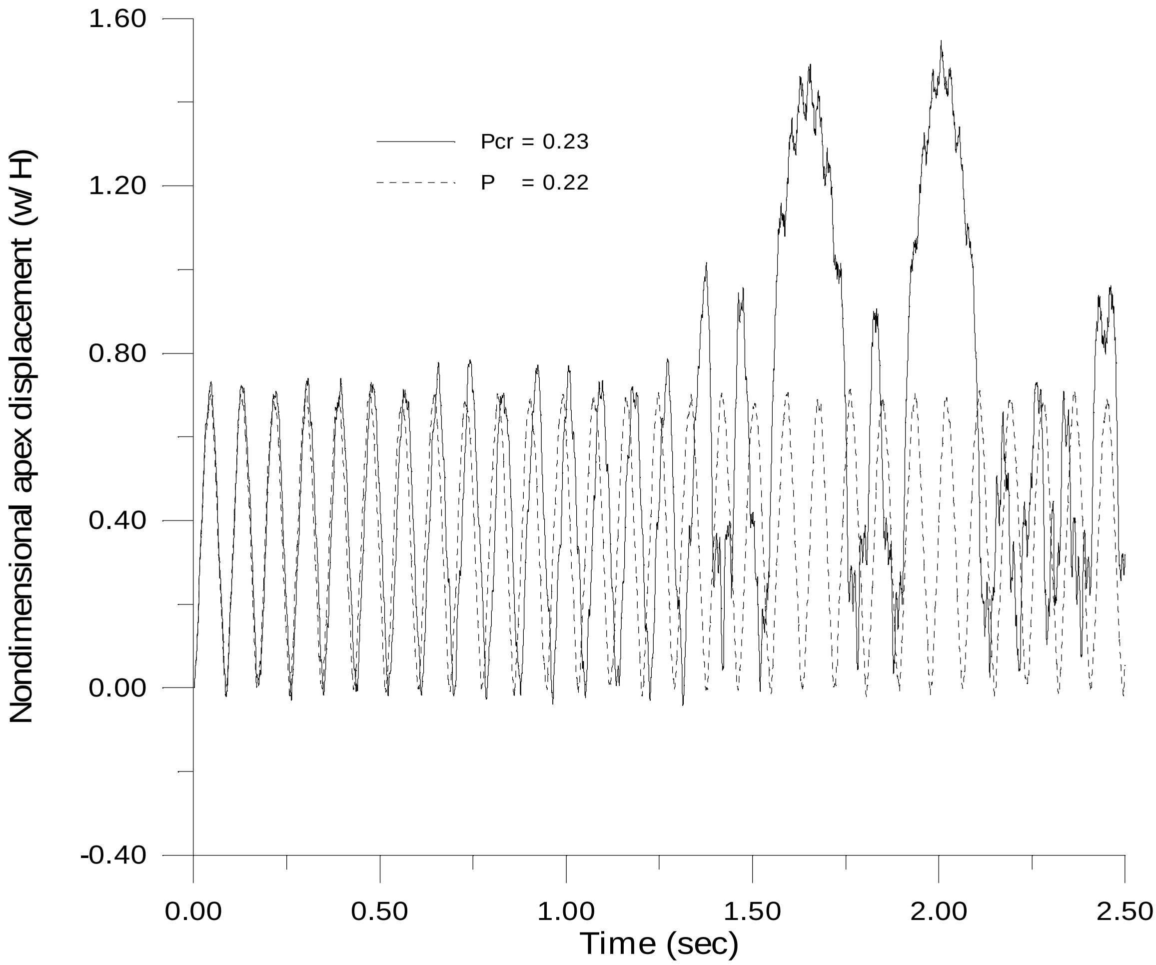

Figure 2. Nondimensional displacement – time curves for λ = 6

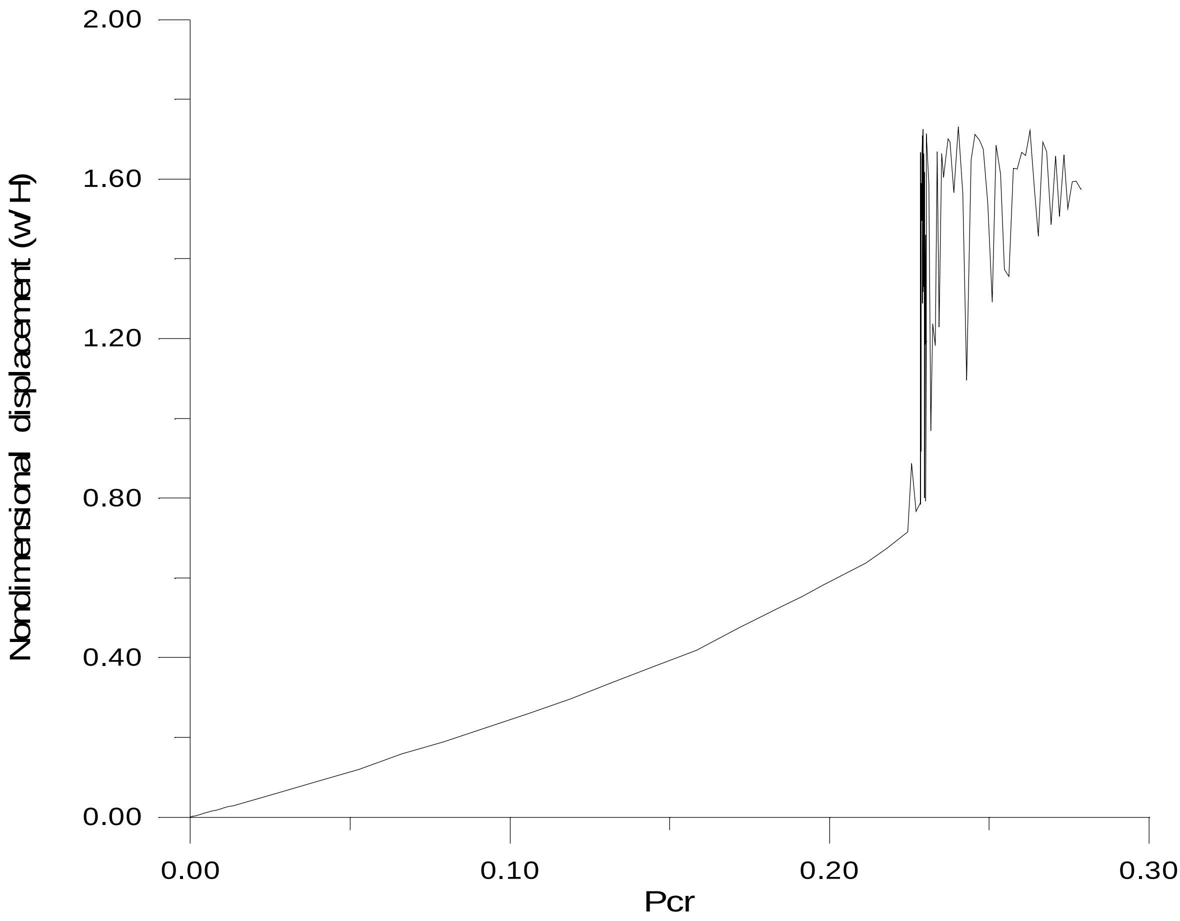

Figure 3. Dynamic buckling analysis of spherical shell for λ = 6.

Conclusions

Finite element incremental formulations for nonlinear analysis of axisymmetric shells of revolution have been reviewed and derived using continuum mechanics principles. The validity of the theoretical procedure developed, the accuracy and applicability of the finite element employed, have been demonstrated. Spherical shells with opening around the apex subjected to peripheral dynamic loading were investigated. It has been observed that dynamic snap-through buckling is also possible under peripheral loading conditions. This phenomenon can be generalized and used in the design of devices such as actuators and transducers.

References

Akkas, N. and Odeh, G., 2001, A Novel Snap-Through Buckling Behavior of Axisymmetric Shallow Shells with Possible Application in Transducer Design. *Comp. Struct*. Volume 79 No. 29-30 pp. 2579-2585.

Akkas, N. and Odeh, G., 2000, A novel snap-through buckling behaviour of axisymmetric shallow shells subjected to peripheral loading. Nha Trang 2000 International Colloquium in Mechanics of Solids, Fluids, Structures and Interactions, pp 562-570 *Nha Trang*, Vietnam 14 – 18 August.

Akkas, N. and Odeh G., 1998, A Novel Snap-through Buckling Behavior of Axisymmetric Shallow Shells with Possible Application in Transducer Design. The Fourth International Conference on Computational Structures Technology, *Advances in Civil and Structural Engineering Computing for Practice*, pp. 247-253. Edinburgh, Scotland 18 - 20 August

Bathe, K. J., 1982, Finite element procedures in engineering analysis. *Prentice-Hall*.

Odeh, G., 1998, Nonlinear Analysis of Shells of Revolution. A Ph. D. Dissertation in Civil Engineering, Middle East Technical University, Ankara, Turkey.

Sugawara, Y., K. Onitsuka, K., S. Yoshikawa, Y., Xu, Q., Newnham, R. E, 1992, Metal-ceramic composite actuators, *J. Am. Ceramic Soc*., V. 75, pp. 996-998.

Zienkiewics, O. C. and Taylor, R. L., 1991, The finite element method. Fourth Edition, Vol. 2, *McGraw-Hill Book Company*.